jet, (which the Zukoski and Spaid model does not) but also holds to the conservation equations. This model is more complex than that of Zukoski and Spaid, and it does require empirical knowledge of the location of the mean height of the Mach disk (y_1), but it is pointed out[1] that y_1 for transverse jets into supersonic streams appears to be well correlated with y_1 for jets discharging into quiescent atmospheres by means of an "effective back pressure" concept, so that the necessary empirical knowledge is, in fact, available for the pressure ratio range of interest. Thus, a model that provides a much more appropriate physical picture of the jet structure and shape, and does not violate continuity, is available and should be preferred by anyone studying jet trajectories.

It is also pointed out in Ref. 1 that anyone wanting a simpler model for predicting either the jet area ratio A_c/A_j (Fig. 3 in Ref. 1) or the location of the outer edge of the Mach disk $\bar{y}$ (discussion of Fig. 17 in Ref. 10 of our article) would do better to use the Newtonian drag model above than the hybrid model of Zukoski and Spaid, and no violation of continuity would arise.

In any case, the authors deemed it important to disclose the fact that the model of Zukoski and Spaid does not preserve continuity, (and therefore, is not consistent with respect to approaches usually taken in fluid mechanics), so that others wanting a model for prediction of jet penetration would be aware of that fact. As the authors noted,[1] the model of Zukoski and Spaid[2] consists of a flow which is initially defined as one-dimensional with all properties given, expanding isentropically, with no mixing, to a downstream station which is again one-dimensional, with flow direction defined and one other property defined, viz. pressure. With these assumptions all properties at the downstream station are also defined by use of the conservation equations, which, of course, include an expression for the momentum flux, which in itself requires the conservation of mass. Moreover, this momentum flux is unique to isentropic flow and for the case under consideration requires a turning force that is not, in general, equal to that which would result from a Newtonian pressure distribution. The cross-sectional area of the flow at the downstream station is also defined; however, the cross-sectional shape and location with respect to the primary stream are not constrained. In Ref. 2, however, this complete definition of flow provided by the isentropic assumption is disregarded and instead, the momentum flux required to turn the isentropically expanding jet is equated to the force of a Newtonian pressure distribution on another body, thus eliminating the possibility of defining a unique control volume.

In their Comment, Zukoski and Spaid also fault the authors for comparing their model with experimental results ("... Billig, et al. chose to interpret a characteristic scale as a physical length e.g., Figs. 14 and 15 of Ref. 5, which is certainly unwarranted, and as expected, gives invalid results"). However, after disclaiming any exact correspondence between features of the flow and their scaling parameter "... the exact correspondence between values calculated from Eq. (3) and any measured feature of the flow, such as penetration height, is purely fortuitous,"[2] they then contradict this disclaimer by concluding that "The agreement between experiment and theory is good over the whole pressure ratio studied, and the dependence on specific heat ratio, molecular weight, and Mach number is correctly predicted."[2] Of course, the whole point of developing analytical models is to be able to correlate experimental results based on some insight into the physical phenomena involved. The authors assert that the model of Ref. 1 provides not only a better representation of the physical phenomena but also better correlation in terms of jet shape (trajectory and representative downstream crossections).

References

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Comments on "Symptomatic Behavior of an Electric Arc with Superimposed Flow"

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THE interest paper by Wutzke, Pfender, and Eckert[1] deals with the experimental and theoretical study on a transition of a d.c. longitudinally blown arc from a stable regime with a fixed anode spot to a regime of an arc end moving in a gas flow when continuous discharge is maintained by repeated breakdowns of a near-electrode gas layer. Theoretically the stability of an arc anode spot with stable discharge is explained by the balance in the near-electrode region between the electromagnetic force which is caused by current interaction with a self-magnetic field and aerodynamic force. The experimental data are correlated by the Reynolds number, in which the length of a radial arc section from an arc column surface to an electrode wall is taken as a scale dimension.

To our point of view, there is some contradiction in such approach. The Reynolds number shows the interaction between the inertia forces of a gas flow and friction forces. The electromagnetic force which is the basis of theoretical consideration was not at all taken into account when correlating experimental data.

The interaction of electromagnetic and aerodynamic forces for the case under investigation may be described in Ref. 2

$$\Pi = (\rho v^2 d^2/4\mu_0 I^2)^{1/2}$$

where ρ is the gas density at $T = 300°K$ and appropriate pressure, v is the velocity, d is the characteristic dimension, I is the current, μ_0 is the magnetic constant.

This number is proportional to the inverse value of the aerodynamic drag coefficient. Probably, when correlating the experimental data it is advisable to consider the number Π as a function of the Reynolds number or of any other numbers which play an essential role in the phenomenon considered. The value of the number Π for the data presented in Ref. 1 varies from 0.6 to 4.0, that is already not bad. The best correlation is given by the dependence of Π upon the Peclet number, in which the blowing arc velocity is expressed through Π

$$Pe = \rho_0 c_{p_0}{}^2 \mu_0 I^2/\lambda_0{}^2$$

Here the scale values of the density ρ_0, specific heat capacity c_{p_0} and thermal conductivity λ_0 are taken according to data in Ref. 3. These are typical for the conditions at the surface of a high-current arc column.

The Peclet number takes into account the dependence of dimensions of a radial arc section upon convective heat transfer at the arc column surface. Heat transfer by conduction inside an arc also lays a certain role, but its influence at the small Peclet numbers is probably less essential. Since the Prandtl number variation is small, then in the correlation $\Pi = f(\text{Pe})$ the influence of friction processes which is represented by the Reynolds number is also taken into account. The absolute values of the Peclet numbers show that stable discharge of an arc is realized mainly in a laminar gas flow near an arc surface.

Figure 1 shows the relation $\Pi = f(\text{Pe})$. It is seen that the correlation appears to be sufficiently good for all gases studied,[1] besides nitrogen. Probably when an arc is blown by nitrogen the electrode jets are more essential than when an arc is blown by other gases, preventing from deflecting near-electrode arc sections

Received December 21, 1971.
Index category: Plasma Dynamics and MHD.
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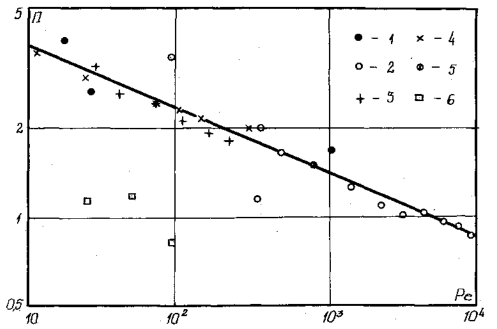

Fig. 1 Dependence of the number Π upon Peclet number: 1—argon, $P = 0.66 \cdot 10^5\ N/m^2$, $d = 5$ mm: 2—argon, $P = 0.13 \cdot 10^5\ N/m^2$, $d = 10$ mm: 3—helium, $P = 0.395 \cdot 10^5\ N/m^2$, $d = 5$ mm: 4—helium, $P = 0.263 \cdot 10^5\ N/m^2$, $d = 10$ mm: 5—hydrogen, $P = 4.6 \cdot 10^3\ N/m^2$, $d = 10$ mm: 6—nitrogen, $P = 6.5 \cdot 10^3$ and $0.12 \cdot 10^5\ N/m^2$: $d = 5$ mm and $d = 10$ mm.

in the direction opposite to gas motion. In addition, in this case local pressures may essentially increase and physical plasma properties may vary. The generalized curve $\Pi = f(\mathrm{Pe})$ may be approximated by the expression

$$\Pi = 6.03\,(\mathrm{Pe})^{-0.21} \tag{1}$$

References

[1] Wutzke, S. A., Pfender, E., and Eckert, E. R. G., "Symptomatic Behavior of an Electric Arc with a Superimposed Flow," *AIAA Journal*, Vol. 6, No. 8, 1968, p. 1474.

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Reply by Authors to O. I. Yasko

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YASKO presents in his discussion the results of Ref. 1 in a dimensionless form. This is generally desirable in continuum mechanics but the authors feel that a few comments are in order regarding its use in the present situation.

Yasko implies that in Ref. 1 the Reynolds number was considered as the only important dimensionless parameter. Actually Figs. 4 and 5 were presented in support of the conclusion that for a specific gas and a constant electric current the Reynolds number correlates the data satisfactorily. The influence of the electric current was demonstrated in Fig. 6. We are, therefore, in agreement with Yasko that the electric current in addition to the Reynolds number influences the flow and energy processes and, therefore, also the restrike behavior of an arc.

Received February 16, 1972.

Index category: Plasma Dynamics and MHD.

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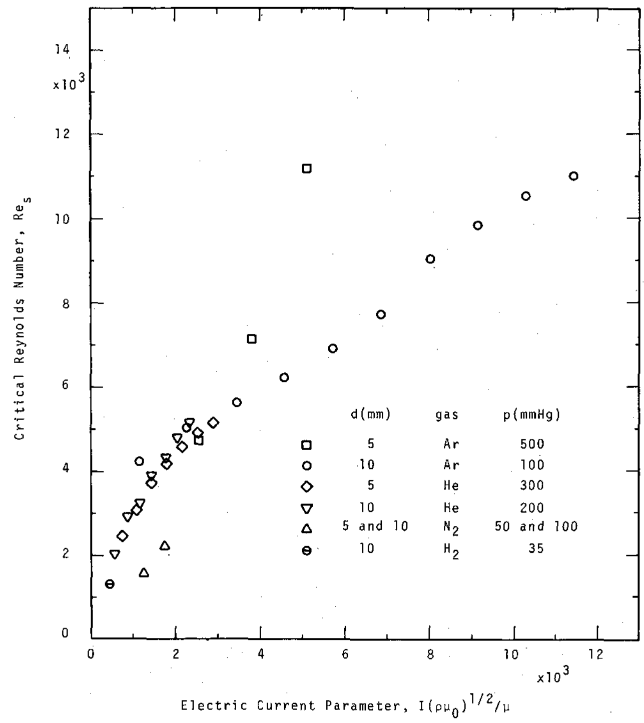

Fig. 1 Dependence of critical Reynolds number on electric current parameter (double anode configuration).

The main advantage obtained by a dimensionless presentation of experimental results lies in the possibility to generalize them. The dimensionless parameters which prescribe the process have then to be based on the boundary conditions or in experimental terms on those physical quantities which can be changed at the desire of the experimenter. In the present situation the following quantities are of this nature: the scale of the experimental setup described by a selected length S, the upstream velocity V, pressure p, and temperature T of the gas as well as its nature (because it determines the way in which the thermodynamic and transport properties involved change with the thermodynamic state) and the magnitude I of the electric current. The following dimensionless parameters can be obtained from the conservation equations: a Reynolds number, $Re = \rho VS/\mu$, a Prandtl number Pr and a parameter $J = I(\rho\mu_0)^{1/2}/\mu$ (the nomenclature is the same as in the paper) which appears more convenient to use than the parameter π in Yasko's discussion because it contains physical properties only in addition to the electric current. The Prandtl number was not varied over a sufficiently large range to establish its influence. The parameters listed above would completely describe the behavior of the arc if the physical properties involved could be considered as constant. In reality, however, they vary considerably and it must be left to experience whether a presentation of test results with these parameters is satisfactory. We have, therefore, replotted the data from Fig. 6 of Ref. 1 in Fig. 1 of this reply. It may be observed that the use of the parameter J instead of the electric current I somewhat improves the correlation.

Dimensionless analysis has the disadvantage that it does not give any information on the details of the physical processes involved. These can be obtained by detailed experimentation or analysis only and the second part of Ref. 1 proposes an analysis based on a model which describes the essential physical processes.

Reference

[1] Wutze, S. A., Pfender, E., and Eckert, E. R. G., "Symptomatic Behavior of an Electric Arc with a Superimposed Flow," *AIAA Journal*, Vol. 6, No. 8, August 1968, p. 1474.